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Hypersonic Flow Past an Axisymmetric Body with Small Longitudinal Curvature

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The problem of steady supersonic flow past an axisymmetric body with small longitudinal curvature is solved by means of a perturbation technique. Assuming a slender body within the framework of hypersonic small-disturbance theory, the resulting differential equations are solved explicitly in closed form using the weak polar crossflow approximation. The perturbation quantities are expressed as series expansions involving powers of the radial distance from the nose of the body and functions of a single angular variable θ. The various flowfield quantities including the surface pressure coefficient have been calculated explicitly in terms of the series. The present analytical results compare favorably with available experimental data and Van Dyke's numerical calculations for hypersonic flow past an ogival-shaped slender body.

Nomenclature

a	= speed of sound
C_p	= pressure coefficient
$f(r)$	= function giving deviation of body shape function from that of a right circular cone
$g(r)$	= function giving deviation of shock shape from a straight oblique shock
G_m	= ratio of perturbations of shock angle and body angle
K_δ	= hypersonic similarity parameters, $M_\infty \delta$
M_∞	= freestream Mach number
m	= power in series expression
$\hat{n}$	= unit normal vector
p	= pressure
P_m	= pressure perturbation function
r,θ,ϕ	= spherical polar coordinates
R_m	= density perturbation function
u,v	= radial and polar velocity components, respectively
U_m, V_m	= radial and polar velocity perturbation functions, respectively
V	= velocity vector
x,R,ϕ	= cylindrical polar coordinates
β	= unperturbed shock angle
γ	= ratio of specific heats
δ	= semivertex angle of basic circular cone
ϵ	= small-perturbation parameter
λ_m	= see Eq. (30)
ρ	= density
ξ_0	= ρ_∞ / ρ_0

Subscripts

∞	= freestream condition
0	= zeroth order
1	= first order
b	= condition on body
m	= mth term in series expression
s	= condition at shock wave

Introduction

THE circular cone is a basic axisymmetric body shape with supersonic flowfield properties that are extensively tabulated.[1] The supersonic flow past a circular cone at small angle of attack is also tabulated and well understood.[2] Supersonic flow past cones that have cross sections which deviate slightly but arbitrarily from a circle have recently been investigated.[3]

The purpose of this work is to develop an approximate analytical solution that illustrates the general flowfield features of supersonic flow past a pointed body that differs from a right circular cone as a result of a small longitudinal curvature. Using a regular perturbation scheme, we seek solutions that differ slightly from a known flowfield, the supersonic flow past a right circular cone. When combined with other solutions which describe the effects of small angle of attack[4] and slight deviation of the cross section from a circle,[3] the flow past rather general body shapes can now be described.

Others have investigated the problem of supersonic flow past a body with longitudinal curvature. Hayes and Probstein[5] describe the well-known empirical tangent cone method. The shock expansion method of Epstein[6] has also been used to analyze such flows. Van Dyke,[7] using the nonlinear hypersonic small-disturbance theory, has analyzed the flow past ogival-shaped bodies. However, none of these earlier approaches are completely analytical. They all require numerical integration of the use of tables. In contrast, the present work yields results in the limit $M_\infty \to \infty$ and $\theta \to 0$ which are given explicitly in closed form. As a consequence, they are easy to use and lend themselves more readily to preliminary design applications.

Analysis

Formulation of Perturbation Problem

In spherical coordinates, as shown in Fig. 1, we represent the pointed axisymmetric body at zero angle of attack that has slight longitudinal curvature by $\theta_b = \delta - \epsilon f(r)$. Here, δ is the semivertex angle of the basic right circular cone and ϵ is a small parameter. The curvature function $f(r)$ is an arbitrary function of the radial distance r and depends upon the given shape of the body. For ϵ small, the body curvature is given by $\epsilon(2f' + rf'')$. The shape of the associated shock wave is similarly represented by $\theta_s = \beta - \epsilon g(r)$, where β is the angle of the shock wave of the basic cone flow and $g(r)$ a function to be determined.

Since the longitudinal curvature is proportional to ϵ, which is assumed small, the various flowfield quantities are expanded in powers of ϵ,

$$V(r,\theta) = V_0(\theta) + \epsilon V_1(r,\theta) + O(\epsilon^2) \qquad (1)$$

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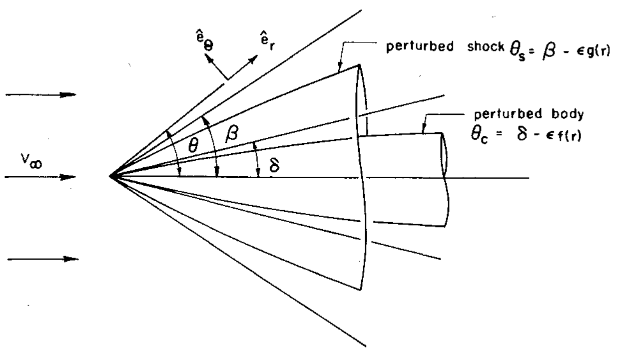

Fig. 1 Geometry of flow.

$$p(r,\theta) = p_0(\theta) + \epsilon p_1(r,\theta) + O(\epsilon^2) \tag{2}$$

$$\rho(r,\theta) = \rho_0(\theta) + \epsilon \rho_1(r,\theta) + O(\epsilon^2) \tag{3}$$

The subscript zero refers to the basic circular cone flow which, being conical, depends only on θ. The subscript 1 quantities are the first-order corrections due to small longitudinal curvature.

The velocity, pressure, and density must satisfy equations expressing conservation of mass, Newton's second law, and the first law of thermodynamics. In this analysis, we assume the flow is inviscid, adiabatic, and steady and the gas is calorically perfect. Substituting Eqs. (1-3) into these governing and equating coefficients of powers of ϵ to zero gives a hierarchy of problems, the first of which describes flow past a right circular cone at zero angle of attack. The second problem describes the first-order effects of small longitudinal curvature. Because the basic cone flow is conical, the equations determining V_1, p_1, and ρ_1 are equidimensional in r and therefore possess power law solutions. It is thus convenient to express the first-order quantities in the form of a series in r,

$$\epsilon u_1(r,\theta) = \Sigma \epsilon_m r^m U_m(\theta) \tag{4}$$

$$\epsilon v_1(r,\theta) = \Sigma \epsilon_m r^m V_m(\theta) \tag{5}$$

$$\epsilon p_1(r,\theta) = \Sigma \epsilon_m r^m P_m(\theta) \tag{6}$$

$$\epsilon \rho_1(r,\theta) = \Sigma \epsilon_m r^m R_m(\theta) \tag{7}$$

where here the sums are taken over all possible values of m. Similarly, the body and shock shape representations are taken to be

$$\theta_b = \delta - \Sigma \epsilon_m r^m \tag{8}$$

$$\theta_s = \beta - \Sigma \epsilon_m G_m r^m \tag{9}$$

The first-order perturbation equations then become four ordinary differential equations for U_m, V_m, P_m, and R_m as functions of θ. The three quantities V_m, P_m, and R_m can be evaluated in terms of U_m and the basic cone solution. The results are

$$V_m(\theta) = \frac{m}{m+1}\frac{1}{\rho_0 v_0} I_m + \frac{1}{m+1}\frac{\mathrm{d}U_m}{\mathrm{d}\theta} \tag{10}$$

$$P_m(\theta) = I_m - \rho_0 u_0 U_m - \rho_0 v_0 V_m \tag{11}$$

$$R_m(\theta) = -\frac{1}{a_0^2}\left[\rho_0 u_0 U_m + \frac{\rho_0 v_0}{m+1}\frac{\mathrm{d}U_m}{\mathrm{d}\theta} - \left(\gamma - \frac{m}{m+1}\right) I_m\right] \tag{12}$$

where the quantity I_m is given by

$$I_m = P_m + \rho_0 u_0 U_m + \rho_0 v_0 V_m \tag{13}$$

The governing equations for P_m, U_m, and V_m can be manipulated to yield an explicit, exact solution for I_m,

$$I_m(\theta) = I_m(\beta)\exp\left[\int_\theta^\beta \frac{m u_0 + \gamma v_0^2 (u_0 + v_0')/a_0^2}{v_0}\,\mathrm{d}\theta\right] \tag{14}$$

The constant $I_m(\beta)$ will be determined later from the boundary conditions. Eliminating V_m and R_m from the mass conservaton equation yields a second-order, linear, ordinary differential equation for U_m which can be written in the form[8]

$$U_m'' + U_m'\cot\theta + U_m L_m(\theta) = F_m(\theta) + H_m(\theta) \tag{15}$$

where

$$F_m(\theta) = \frac{m I_m}{\rho_0 v_0^2}(m u_0 + v_0' - v_0\cot\theta) + \frac{I_m}{\rho_0 a_0^2} \times \Big\{(\gamma(m+1) - m)\Big[2v_0\frac{\mathrm{d}}{\mathrm{d}\theta}\ln(v_0/a_0) - v_0\cot\theta - u_0(m+2) + v_0' + m u_0 + \gamma v_0^2(u_0+v_0')/a_0^2\Big] + \gamma m(u_0 + v_0')\Big\} \tag{16}$$

$$H_m(\theta) = A_1(\theta)U_m'' + A_2(\theta)U_m' + A_3(\theta)U_m \tag{17}$$

$$A_1(\theta) = v_0^2/a_0^2 \tag{18}$$

$$A_2(\theta) = +v_0[v_0\cot\theta/a_0 + (2m+3)u_0/a_0]/a_0 - (1 - v_0^2/a_0^2)\frac{\mathrm{d}\ln\rho_0}{\mathrm{d}\theta} - 2v_0^2\left(\frac{\mathrm{d}\ln a_0}{\mathrm{d}\theta} - \frac{\mathrm{d}\ln v_0}{\mathrm{d}\theta}\right)\Big/a_0^2 \tag{19}$$

$$A_3(\theta) = (m+1)[-(2-\gamma)u_0^2 v_0' v_0/a_0^4 + v_0^2(1 - (2-\gamma)u_0^2/a_0^2)/a_0^2] \tag{20}$$

$$L_m(\theta) = (m+1)[(m+2)(1 - u_0^2/a_0^2) - u_0 v_0(\cot\theta + v_0'/v_0)/a_0^2] \tag{21}$$

Boundary Conditions

The boundry conditions for this problem are the shock jump relations and the condition of zero mass flux through the body—the so-called tangency condition. When the perturbation expansions [Eqs. (4-7)] are substituted into the shock jump relations and the values at θ_s are transferred to the basic shock $\theta = \beta$ by means of a Taylor series expansion, the following results are obtained

$$U_m(\beta) = V_\infty \sin\beta(1+m)(1-\xi_0)G_m \tag{22}$$

$$V_m(\beta) = \{u_0 v_0 U_m/a_0^2 - \gamma v_0 I_m/\rho_0 a_0^2 + \xi_0 V_\infty \cos\beta[(1-\xi_0)m + 1]G_m + (\rho_0 v_0)' G_m/\rho_0\} \div (1 - v_0^2/a_0^2) \tag{23}$$

$$P_m(\beta) = -\rho_\infty V_\infty^2 \cos\beta\sin\beta(1-\xi_0)(1+m)G_m - \rho_0 v_0 V_m \tag{24}$$

$$R_m(\beta) = -\rho_0 V_m / v_0 + \rho_\infty V_\infty \cos\beta [(1-\xi_0)m+1] G_m + (\rho_0 v_0)' G_m / v_0 \tag{25}$$

where $\epsilon_0 = \rho_\infty / \rho_0(\beta)$. We also note that $I_m(\beta)$ can be determined from Eqs. (22-24).

The tangency condition at the body can be used to show that

$$V_m(\delta) = v_0' - m u_0 \tag{26}$$

Weak Polar Crossflow Approximation

We now wish to simplify the governing differential equation (15) by adopting the weak polar crossflow approximation. The term $(v_0/a_0)^2$ varies from a maximum at the shock to zero on the body,

$$0 \le \frac{v_0^2}{a_0^2} \le \frac{v_0^2}{a_0^2}\bigg|_{\text{shock}} = \frac{(\gamma-1)M_\infty^2 \sin^2\beta + 2}{(\gamma+1)M_\infty^2 \sin^2\beta - \gamma + 1} \tag{27}$$

For $M_\infty \sin\beta$ large, the upper bound becomes $(\gamma-1)/2\gamma(=1/7$ for $\gamma = 1.4)$. Ignoring terms of order v_0/a_0 in Eq. (15) yields a form that can be solved analytically in terms of known functions. Rasmussen and Lee[9] have shown that the hypersonic flow past cones of small cross-sectional ellipticity, ignoring terms of order v_0/a_0, gives results that are accurate over the entire range of values of the hypersonic small-disturbance parameter $M_\infty \delta$, especially for values of $M_\infty \delta$ exceeding unity.

In this weak polar crossflow limit $(v_0/a_0 \to 0)$, the density ρ_0 and sound speed a_0 vary slightly across the shock layer and can be accurately approximated by their values at the shock wave. In this case, Eq. (15) reduces to

$$U_m'' + \cot\theta U_m' - \lambda_m^2 U_m = J_m \tag{28}$$

where J_m and λ_m^2 are given by

$$J_m = \frac{I_m(\beta)}{\rho_0 v_0^2} \exp\left[\int_\theta^\beta \frac{m u_0}{v_0} d\theta\right] m[(m+2)u_0 + 2v_0'] \tag{29}$$

$$\lambda_m^2 = (1+m)(-m-2+m u_0^2/a_0^2) \tag{30}$$

Hypersonic Small-Disturbance Approximation

Equation (28) is not yet in a form that can be integrated in terms of known functions. However, this limitation can be overcome if we consider slender bodies $(\theta \to 0)$ at high Mach numbers $(M_\infty \to 0)$ such that the hypersonic similarity variable $K_\theta \equiv M_\infty \theta$ remains finite. This limit defines the hypersonic small-disturbance approximation for which the basic right circular cone flow can be accurately approximated by[10]

$$u_0/V_\infty = 1 - \frac{\delta^2}{2}[K_\delta^2/K_\delta^2 + 2\ln(K_\beta/K_\theta)] \tag{31}$$

$$v_0/V_\infty = -\theta[1-\delta^2/\theta^2] \tag{32}$$

The shock angle β and cone angle δ are related by

$$\beta/\delta = [(\gamma+1)/2 + 1/K_\delta^2]^{1/2} \tag{33}$$

Also $\xi_0 = 1 - \delta^2/\beta^2$. In this case λ_m is a constant and Eq. (28) reduces to a nonhomogeneous Bessel equation, the solution of which is

$$U_m(\theta) = a_1 I_0(\lambda_m\theta) + a_2 K_0(\lambda_m\theta) - I_0(\lambda_m\theta)\int_\theta^\beta \eta K_0(\lambda_m\eta) \times J_m(\eta)d\eta + K_0(\lambda_m\theta)\int_\theta^\beta \eta I_0(\lambda_m\eta) J_m(\eta)d\eta \tag{34}$$

where I_0 and K_0 are modified Bessel functions of the first and second kind of zeroth order. The integrations in Eq. (34) can be carried out and, after considerable algebra, their results are

$$\int_\theta^\beta \eta K_0(\lambda_m\eta) J_m(\eta)d\eta = \left[\frac{\eta^2}{\eta^2-\delta^2} K_0(\lambda_m\eta) I_m(\eta) + \frac{\lambda_m\eta}{m} K_1(\lambda_m\eta) I_m(\eta)\right]_\theta^\beta \tag{35}$$

$$\int_\theta^\beta \eta I_0(\lambda_m\eta) J_m(\eta)d\eta = \left[\frac{\eta^2}{\eta^2-\delta^2} I_0(\lambda_m\eta) I_m(\eta) - \frac{\lambda_m\eta}{m} I_1(\lambda_m\eta) I_m(\eta)\right]_\theta^\beta \tag{36}$$

The constants of integration a_1 and a_2 can be evaluated from the boundary conditions as

$$a_1 = \frac{1}{b_1}\left[K_1(\lambda_m\delta) U_m(\beta) + (m+1) b_2 \frac{K_0(\lambda_m\beta)}{\lambda_m} - (m+1)(m+2) V_\infty \frac{K_0(\lambda_m\beta)}{\lambda_m}\right] \tag{37}$$

$$a_2 = \frac{1}{b_1}\left[I_1(\lambda_m\delta) U_m(\beta) - (m+1) b_2 \frac{I_0(\lambda_m\beta)}{\lambda_m} + (m+1)(m+2) V_\infty \frac{I_0(\lambda_m\beta)}{\lambda_m}\right] \tag{38}$$

where

$$b_1 = I_0(\lambda_m\beta) K_1(\lambda_m\delta) + K_0(\lambda_m\beta) I_1(\lambda_m\delta) \tag{39}$$

$$b_2 = \frac{\delta^4}{\beta^4}\lambda_m\beta G_m V_\infty \left\{\frac{m\beta^2}{\beta^2-\delta^2}[I_1(\lambda_m\delta)K_0(\lambda_m\beta) + K_1(\lambda_m\delta) I_0(\lambda_m\beta)] + \lambda_m\beta[I_1(\lambda_m\delta)K_1(\lambda_m\beta) - K_1(\lambda_m\delta) I_1(\lambda_m\beta)]\right\} \tag{40}$$

and

$$U_m(\beta) = V_\infty \beta(m+1)\frac{\delta^2}{\beta^2} G_m \tag{41}$$

$$I_m(\theta) = \rho_0 V_\infty^2 \beta \frac{\delta^4}{\beta^4}(1+m) G_m \left(\frac{\theta^2-\delta^2}{\beta^2-\delta^2}\right)\left(\frac{m}{2}\right) \tag{42}$$

The remaining constant G_m, the ratio of the perturbations of the shock and body angles, is calculated from Eqs. (12) and (25). The result is

$$G_m = -\left[1-\beta^2\frac{V_\infty^2}{a_0^2}\xi_0^2\right] b_3 \div \left[(m+2)\frac{\delta^2}{\beta^2} + (m+1)\beta^2\frac{V_\infty^2}{a_0^2} \times \xi_0 \frac{\delta^2}{\beta^2}\left(1-\gamma\frac{\delta^2}{\beta^2}\right) + b_4\left(1-\beta^2\frac{V_\infty^2}{a_0^4}\xi_0^2\right)\right] \tag{43}$$

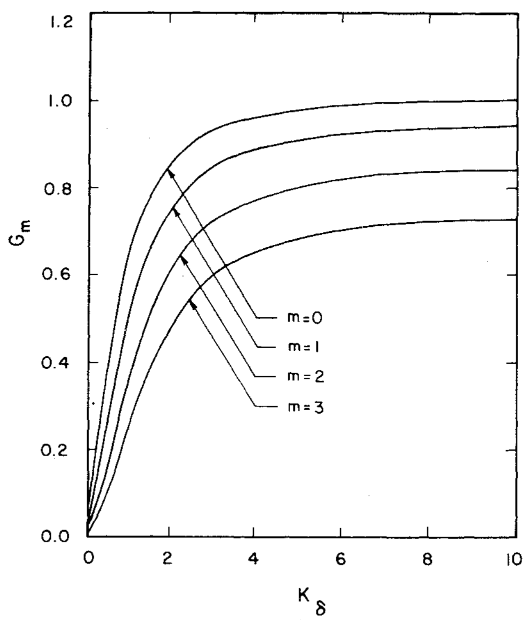

Fig. 2 Ratio between shock and body perturbation G_m as a function of K_δ for various m ($\gamma = 1.4$).

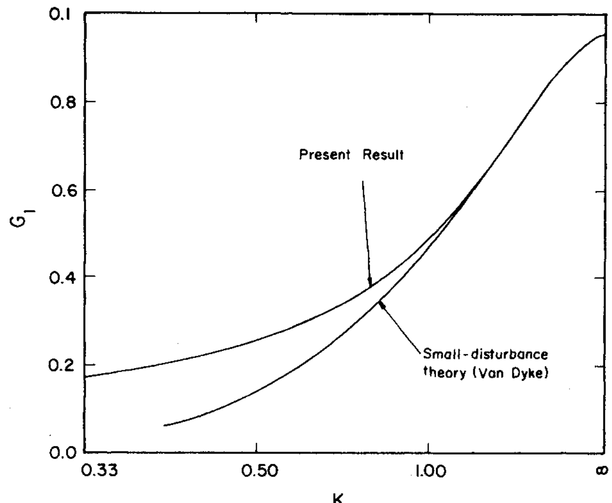

Fig. 3 Comparison between present result for G_1 and numerical result from hypersonic small-disturbance theory due to Van Dyke[7] ($\gamma = 1.405$).

where

$$b_3 = -\frac{(m+2)V_\infty}{\lambda_m\beta[I_0(\lambda_m\beta)K_1(\lambda_m\delta) + K_0(\lambda_m\beta)I_1(\lambda_m\delta)]} \tag{44}$$

$$b_4 = \lambda_m\beta\frac{\delta^2}{\beta^2}\xi_0\frac{I_1(\lambda_m\beta)K_1(\lambda_m\delta) - K_1(\lambda_m\beta)I_1(\lambda_m\delta)}{I_0(\lambda_m\beta)K_1(\lambda_m\delta) + K_0(\lambda_m\beta)I_1(\lambda_m\delta)} \tag{45}$$

Since $V_\infty^2\beta^2/a_0^2$ can be expressed as

$$\frac{V_\infty^2\beta^2}{a_0^2} = \frac{[(\gamma+1)K_\delta^2+2]^2}{2[(\gamma-1)K_\delta^2+2](\gamma K_\delta^2+1)} \tag{46}$$

we see that G_m is a function of K_δ, γ, and m.

Results for G_m as a function of K_δ for $\gamma = 1.4$ and various m are shown in Fig. 2. We see there that G_m has the correct limiting value of linearized theory (e.g., $G_m = 0$) when $K_\delta \to 0$. Also, as $K_\delta \to \infty$, a hypersonic limiting value is achieved. In the Newtonian limit ($K_\delta \to \infty$ and $\gamma \to 1$), we obtain $G_m = 1$, as expected.

It is important to understand that these two limits ($K_\delta \to 0$ and $K_\delta \to \infty$) are obtained correctly in the present theory because the weak polar crossflow approximation is applied only to the governing equation for U_m [Eq. (15)] and not to the boundary conditions. The boundary conditions are satisfied exactly, within the framework of the small-perturbation approximation. As a consequence, the solution achieves both the hypersonic ($K_\delta \to \infty$) and linear ($K_\delta \to 0$) limits correctly and, in general, gives a good approximation over the entire range of K_δ.

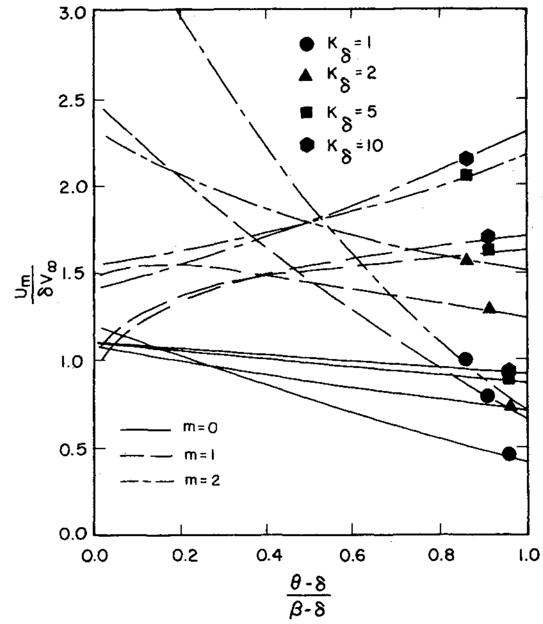

Fig. 4 Radial velocity function U_m ($\gamma = 1.4$).

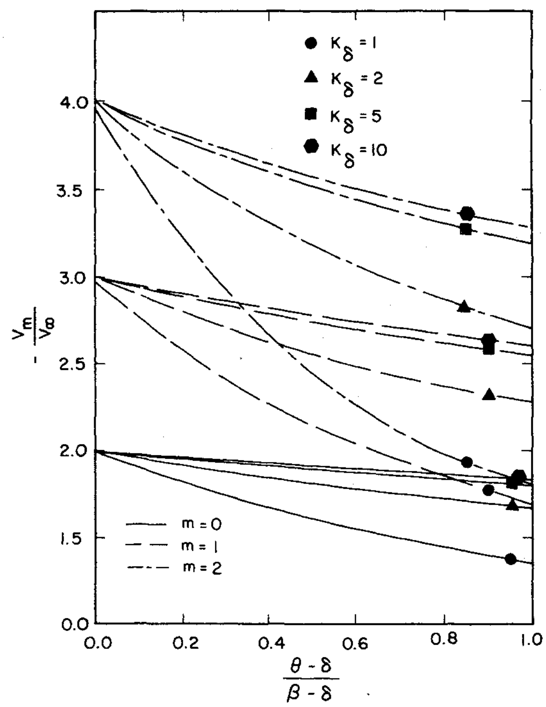

Fig. 5 Polar velocity function V_m ($\gamma = 1.4$).

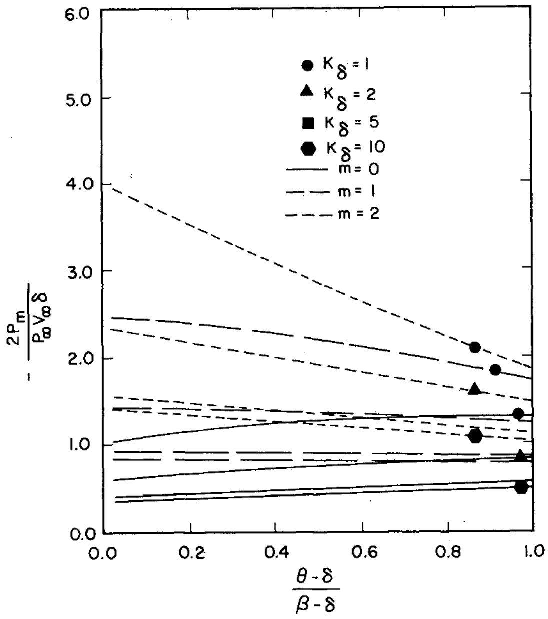

Fig. 6 Pressure function $P_m(\gamma = 1.4)$.

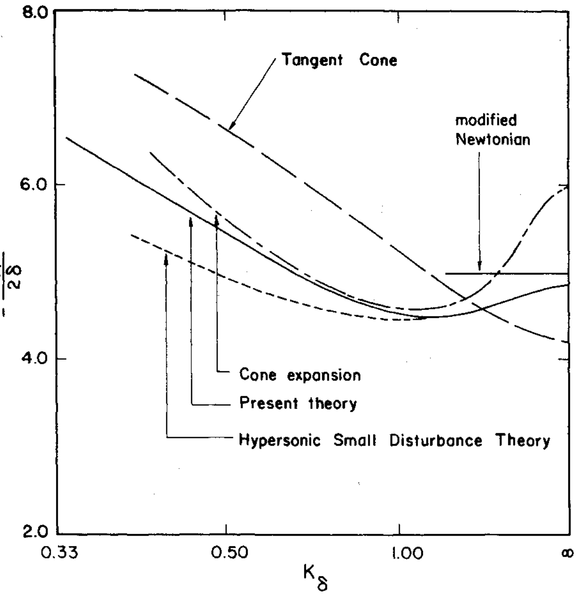

Fig. 8 Initial pressure gradient on an ogival body ($\gamma = 1.405$).

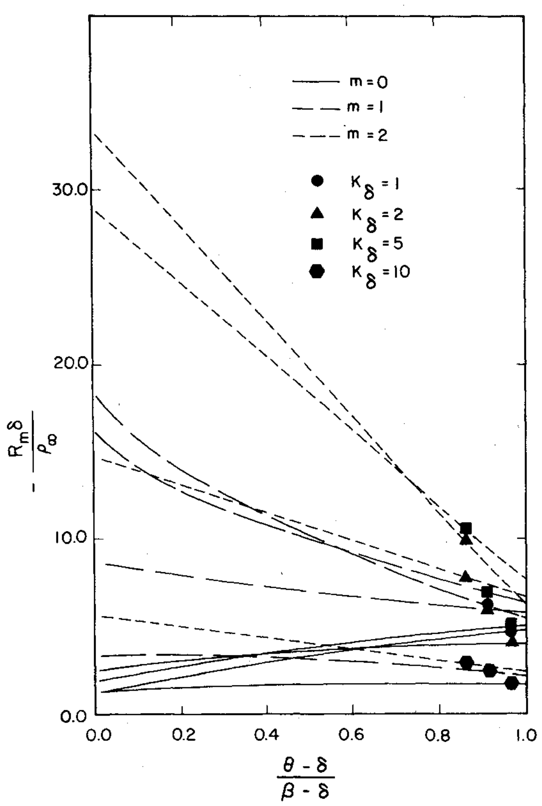

Fig. 7 Density function $R_m(\gamma = 1.4)$.

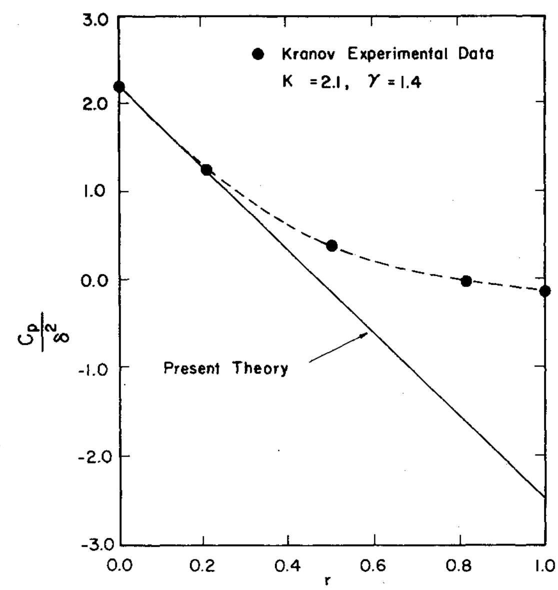

Fig. 9 Surface pressure on the ogive $\theta_b = \delta(1 - r/2)$—comparison of theory and experiment.

Figure 3 shows a comparison between the present result for G and that obtained by Van Dyke[7] using nonlinear hypersonic small-disturbance theory. As is evident, the comparison is very good, especially when K_δ is greater than unity.

The pressure P_m, density R_m, and polar velocity V_m can be determined from Eqs. (10-12). The flowfield variables U_m, V_m, P_m, and R_m are shown as functions of $\theta' = (\theta - \delta)/(\beta - \delta)$ for various m and K_δ in Figs. 4-7 for $\gamma = 1.4$.

Surface Pressure Coefficient

The surface pressure coefficient C_p can be written

$$C_p = C_{p0} + \Sigma \epsilon_m r^m C_{pm} + O(\epsilon^2) \qquad (47)$$

where

$$C_{pm} = -2\rho_0 V_\infty U_m(\delta)/\gamma M_\infty^2$$

The velocity perturbation $U_m(\delta)$ can be calculated from Eq. (34); C_{p0} is known from the basic cone solution. The present result for C_{pm} agrees exactly with linearized theory in the limit $K_\delta \to 0$ and with modified Newtonian theory (e.g., Newtonian plus Busemann correction) in the double limit $K_\delta \to \infty$, $\gamma \to 1$.

Figure 8 shows a comparison of the present result with those of other methods for the initial pressure gradient on an ogival body, $-C_{pl}/2\delta$. The present results again agree well with the numerical calculation of Van Dyke[7] when $K_\delta > 1$.

Finally, Fig. 9 shows a comparison of the results of the present theory with experimental data[11] for the surface pressure coefficient on an ogival body given by $\theta_b = \delta(1 - r/2)$. These results show the present theory works well, provided the body angle perturbation (here $\delta r/2$) is small compared to δ; that is, if $r/2$ is small. Figure 9 shows that this condition is satisfied if $r < 0.2$.

Conclusion

Approximate flowfield results for the supersonic flow past an axisymmetric body which has slight longitudinal curvature have been obtained explicitly in closed form. The results appear to be accurate when the perturbation of the body angle is less than 10% of the body angle of the basic right circular cone. The present results are useful over the entire range of K_δ from the linearized theory limit ($K_\delta \to 0$) to the hypersonic limit ($K_\delta \to \infty$) and are especially accurate for $K_\delta > 1$.

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